Impact of a Monetary Union on Sovereign Credit Ratings

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ABSTRACT

The current European debt crisis has made sovereign credit risk a popular topic. In this paper we adapt an established structural model for assessing sovereign credit risk and expand it to evaluate the cases of California and Greece. Specifically, major political events such as a bailout or a breakout from a monetary union are not accounted for in current models despite that they may introduce non-linearities in the behavior of the default probability. In this paper, we attempt to account for these extra factors and solve the problem numerically. We rely on a 2-D finite differences method, modeling both the risky assets of our target sovereignty and the ones of its encompassing monetary union. Finally, we detail a method for hedging sovereign credit risk using tradable securities.

Section 1

Introduction

We base our formulation on the model proposed by the paper "Contingent Claims Approach to Measuring and Managing Sovereign Credit Risk" (Gray et al. 2007). This work details a method for backing out the implied value of an entity's unobservable sovereign assets using a contingent claims pricing methodology. This approach treats an entity's local currency liability as a call option on a sovereignty's assets (similar to treating equity as a call option on corporate debt), and the foreign currency liability as a risk-free asset minus a put option on the assets. Here, local currency and foreign currency liabilities are observable and the sovereign assets can be backed out using the Black Scholes Merton formula for option pricing. Then, the probability of default is modeled using a Merton-style credit risk model.

Our first contribution is to adjust the concept described above to a case where the sovereignty is positioned within a monetary union (as such, within this paper, we will often refer to our target entity as the "state" and the encompassing monetary union as the "union"). We make the alteration because the original model is not directly applicable to the cases of California and Greece, the two entities on which this paper focuses; specifically, being part of a monetary union (USA and EMU, respectively) relinquishes their sovereign control on the local currency. To take this into account, we first adapt the problem by calculating the value of the assets of the union and divide the result among the state entities based on their percentage of total GDP. This gives us a consistent method of quantitatively calculating the value of the assets for California and Greece. We will then use a Merton Model framework for calculating the state's probability of default.

The first question to be posed is whether it is possible for a state to default without the union doing so. There are very few examples of this happening in history, though such events have occurred in the past; Orange County's default could be considered such a case. Also Greece's recent renegotiation of its debt with private creditors, while may not be a default technically, could be considered to be a credit event. This notion forces the modeler to consider the question of what would cause a union to intervene to save the state from defaulting. One approach is to assume that the union would intercede when there is risk of contagion that would cause greater economic difficulties across its domain. Then, theoretically a union will only let the state default when the rest of the its realm is "healthy" and far from the danger of defaulting. In our model, this notion will take the form of barrier on the union's asset process; that is, when its asset value falls below the barrier, the union will start to "bail out" the state entities so as to prevent contagion.

A second feature we address is that of a fiscal coalition. In our model, if the union issues debt, we split the liability among the state entities based on their percentage of total assets. Hence, if the state's economy is suffering relative to its counterparts in the union, its portion of the union's debt is smaller, thereby reducing the state's default barrier. Although there has been discussion around the issuance of Euro Bonds and other similar instruments, the Euro Area does not have fiscal unity. As a result, it is conceivable that a state would exit the union to gain more control over its monetary policy tools. However, when there is an explicit fiscal confederacy that issues debt, it is much more difficult for the state entity to split off from the union. Therefore, in such situations, we assume that the probability of a state entity breaking away for a union is zero (for example, it is extremely unlikely that California will leave the United States). In this paper, we implement the idea by modeling both the union and state asset processes in concert. If there is no fiscal association, our model allows for the state to break away from the union; this then changes the amount of the union's assets as the state's breakout takes a portion of the union's wealth.

The model in this paper is designed to provide an estimate of a state entity's probability of default. While we do not claim that its output provides a perfect appraisal of the default likelihood, we do contend that it offers a relative strength metric. Once generated for a number of entities, the model's outputs can be split into deciles and used to determine relative credit ratings indexed between one and ten. The model setup follows in Section 2.

Finally, in order to hedge against default, we take advantage of the option pricing framework of our model by constructing a delta hedge. In order to do this, we replicate the assets of the sovereignty by using a basket of liquid, easily tradable underlying securities using a Kalman Filtering technique.

SECTION 2

UNOBSERVABLE SOVEREIGN ASSETS

Although information of various categories on government debt is typically available from a sovereignty's central bank or Department of Treasury database, estimating the government's asset value is significantly more formidable. Gray et al. (2007) develop a novel balance sheet approach to obtain the sovereign's implied asset value (A) and implied asset volatility (σ_A) by using the Merton (Merton, December 1973) structural model; we adopt the same methodology here for both sovereign and municipal entities. The government debt issued in domestic currency (L) has equity-like features as it acts as a junior claim to the debt issued in foreign currencies. For the case of a municipal government, we delineate its debt into subordinate (junior) and senior levels according to their definition at issuance.

The modified Merton model approach (Gray et. al, 2007) relates asset value (A) and level of junior debt (L) in the following equations:

$$L = A \cdot N(d_1) - B_f e^{-rT} N(d_1 - \sigma_A \sqrt{T})$$

and

$$L\sigma_L = A\sigma_A N(d_1)$$

$$\text{where } d_1 = \frac{\ln \frac{A}{B_f} + \left(r + \frac{1}{2}\sigma_A{}^2 \right) T}{\sigma_A \sqrt{T}}$$

In this set of equations, B_f, the strike price, is a state government's senior debt whose value can be found either on the government's website or a financial database like Bloomberg. In the case of a sovereign government, the senior debt level is the sum of those debts issued in foreign currencies. In addition, r is the US 1-year Treasury rate, obtained from St. Louis Fed (FRED); $T = 1$ year is the time horizon under consideration in our model; Junior debt (equity) volatility (σ_L) is calculated from the time series data of L. By solving the above equations, we can extract the implied asset value A and asset volatility (σ_A) of each government entity. Lastly, it is worth noting that another approach to back out the implied government asset is the Vasicek-Kealhofer model (Bohn & Stein, 2009); this method solves for the asset value and volatility in a recursive fashion.

SECTION 3

2D MODEL SETUP

This section discusses our model's implementation. Since the first Merton model, academic literature has been thorough in dealing with corporate credit ratings, improving the model's specifications and increasing its flexibility through time (e.g. (Black & Cox, 1976), (Longstaff & Schwartz, 1995), (Leland, 1994), etc.). Gray's paper in 2007 introduces a structural model to study sovereign debt credit risk. Our approach aims at adding a new dimension to the problem by incorporating an overseeing "union" entity. This entity can be a monetary union (e.g. Eurozone), political and fiscal union (e.g. USA), or a supranational organization (e.g. IMF).

We model the interactions between the assets divided by liabilities (as defined in the previous section) of both the **S**tate (the smaller entity, denoted by the letter S) and the **U**nion (the higher entity denoted by the letter U).

Both assets are considered to follow Geometric Brownian Motion processes, with correlation between them represented by coefficient ρ. The process drifts, μ_S and μ_U, and volatilities, σ_S and σ_U, respectively, are considered to be constant. Thus, we can write the processes as:

$$\frac{dS}{S} = \mu_S dt + \sigma_S dW_S \quad and \quad \frac{dU}{U} = \mu_U dt + \sigma_U dW_U$$

with $dW_S dW_U = \rho dt$

Our goal is to derive a diffusion equation for the probability of *survival*, $p(S, U, t)$, directly related to the probability of default by $PoD = 1 - p$. The idea is to value an Arrow-Debreu security paying \$1 at the end of the horizon if the state does not default. Since we are modeling a probability, we take it to have no drift.

Using Ito's formula, we can derive a PDE for this probability:

$$\frac{dp}{dt} + \mu_S S \frac{dp}{dS} + \mu_U U \frac{dp}{dU} + \frac{1}{2}\sigma_S^2 S^2 \frac{d^2p}{dS^2} + \frac{1}{2}\sigma_U^2 U^2 \frac{d^2p}{dU^2} + \rho\sigma_S\sigma_U SU \frac{d^2p}{dSdU} = 0$$

To gain additional accuracy around the 0 level, we use the log-processes:

$$\frac{dp}{dt} + \left(\mu_S - \frac{\sigma_S^2}{2}\right)\frac{dp}{dlogS} + \left(\mu_U - \frac{\sigma_U^2}{2}\right)\frac{dp}{dlogU} + \frac{1}{2}\sigma_S^2\frac{d^2p}{dlogS^2}$$

$$+\frac{1}{2}\sigma_U^2\frac{d^2p}{dlogU^2} + \rho\sigma_S\sigma_U\frac{d^2p}{dlogSdlogU} = 0$$

To evaluate the PDE, we use a 2-dimensional finite differences solver, with a Crank-Nicolson scheme for 2nd order accuracy.

A crucial issue is how to deal with the barriers. While the original Merton paper (Merton, December 1973) diffuses the probability and takes the default only at the end of the horizon, the Black-Cox paper (Black & Cox, 1976) assumes a first passage absorbing barrier for default. This is the approach we consider in our work: each time the asset level crosses the default level, the value of the AD security jumps to zero. This can happen for both the state and for the union. If the union defaults, the state is subsequently considered in default.

The added value of our work comes from the scenarios that we consider plausible when interacting within a union:

1. The state can be bailed out by the union: the union owns an option on this probability of default, where the strike is the probability of default of the union at the same time. In other words, when the state is bailed out, it becomes a subsidiary of the union, and thus behaves like the union. This is modeled by a jump to a 1-dimensional PDE considering only the union. We term this event “bailout”.

2. The state can break out of the union: the state owns an option on the probability of default, where the strike is the probability of default of the state alone. In other words, when the state breaks out, it does not depend anymore on the union and behaves by itself. This is modeled by a jump to a 1-dimensional PDE considering only the state. We term this event “breakout”.

Let PoS be the probability of survival; then, we can write our model using 4 finite difference equations: two 2D processes for $PoS_S(S,U)$ and $PoS_U(S,U)$, and two additional 1D processes for $PoS_S(S)$ and $PoS_U(U)$, which describe the behavior after a breakout and bailout, respectively. Their relation can be described as follows.

The first scenario is based on a choice made by the union to bail out the state.

$$PoS_U(S,U) \to \max(P1, P2)$$

$$P1 = PoS_U(S,U) \times PoS_s(S,U) + PoS_U\left(S, U \times \frac{Assets_U - Assets_S}{Assets_U}\right) \times \left(1 - PoS_S(S,U)\right)$$

$$P2 = PoS_U\left(U \times \frac{Debt_U}{Debt_S + Debt_U}\right)$$

The PoS with only one argument is in the case of a bailout when the model jumps to a 1-dimensional process with only the union.

The impact on the PoS for the state is:

$$PoS_S(S,U) \to \begin{Bmatrix} PoS_S(S,U) & if\ P1 > P2 \\ P2 & otherwise \end{Bmatrix}$$

We can see that in this case, since the state does not have the power of decision for the option, it may find its exercise sub-optimal.

A similar jump between equations is done in the case of the breakout scenario. In this case though, the decision is made by the state. If it chooses to breakout, the state becomes driven by its own PDE in a 1-dimensional process. However, with this, it gives up the possibility of benefitting from the bailout of the union.

$$PoS_S \to \max(Q_1, Q_2)$$

$$Q_1(S,U) = PoS_S(S,U)$$

$$Q_2(S,U) = PoS_S(S)$$

In this case, the union has an adjusted default probability that depends on whether the option is triggered; given this, the union no longer depends on the state.

$$PoS_U(S,U) \to \begin{Bmatrix} PoS_U(S,U) & if\ Q_1 > Q_2 \\ PoS_U\left(U \times \frac{Assets_U - Assets_S}{Assets_U}\right) & otherwise \end{Bmatrix}$$

The finite differences framework allows for us to effectively model the possibility that these events occur (which we model as jumps between PDEs). As mentioned in Section 1, we reduce the probability of the breakout scenario in the case of a state within the USA for example, where it is highly unlikely that a state will leave the union.

PREDICTION

Our model can predict effectively the change in the rating by rerunning our model with the same time horizon, and initializing both variables at their expected value ($e.g.\ E(U) = \exp(\mathrm{rT})\, U$).

RESULTS

The finite differences method is applied with a time horizon of 1 year in order to be consistent with the solution proposed by Gray et al. (2007).

In executing our model, we obtain the following map for the probability of survival. It accounts for the possibility of a bailout by the union and breakout by the state. Adjacent to the output, we display a "control" graph, where we do not activate the bailout and breakout scenarios. These graphs represent the probability of survival of the state and are created for illustration, not accounting for real data.

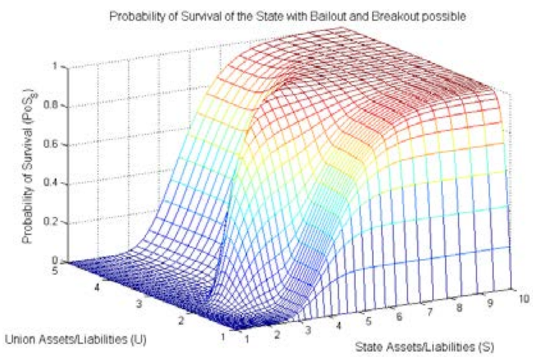

FIGURE 1 - WITH BAILOUT AND BREAKOUT

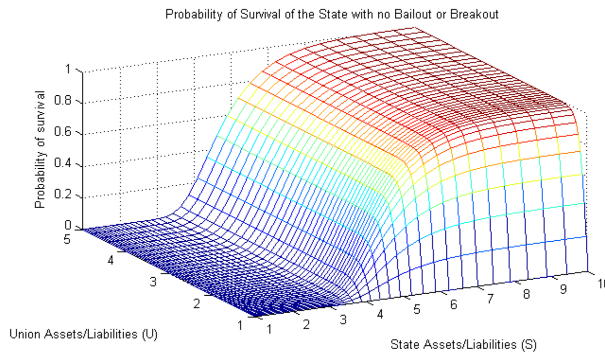

FIGURE 2 - WITHOUT BAILOUT OR BREAKOUT

By comparing both charts, we can see that the bailout has a positive effect on the PoS of the state; this feature causes for a "bump" to appear. This effect indicates that in the case of a bailout, the state can increase its probability of survival. We highlight the significance of the fact that the bump in Figure 1 appears close to the coordinates (2,2). This indicates that the bailout occurs not only if the state is close to default, but also when the union is sufficiently far from default (the region closer to union assets/liabilities = 1 does not have a bailout). However, if the union does very well (assets/liabilities over 3), it does not fear contagion, and does not bail out the state. This can be explained by the fact that the probability of survival of the union is not affected enough by the default of the state in this region to generate a bailout.

The breakout also provides the state with less chance to default when the union goes bankrupt. The effect is less visible, because it only improves the probability of survival for the state when it is healthy, and thus already close to 1.

Once this model is set up, we can use the probability of survival output to calculate the probability of default as

$$PoD = 1 - PoS$$

Then, having done this calculation for many sovereign states, we can provide relative rankings of credit risk. By using deciles, we can develop ratings indexed from 1 to 10.

SECTION 4

HEDGING

Since our model is based on an option-pricing structural framework, we can think of hedging the risk of default in the same way as hedging the sensitivity of an option. In this case, we only hedge the delta exposure as the granularity of our model does not allow for us to gain significant benefit from hedging higher orders.

For a standard delta hedging strategy, we would invest in $\frac{\Delta * Notional}{Asset\ Price}$ number of contracts. However, in our case, our assets are calculated implicitly and are not tradable; thus, we cannot use them to directly delta-hedge our default risk. However, we can assume that the assets of a country are a component of buying and selling various goods; we can then try to replicate a state's assets via the model:

$$Country\ Asset\ Returns = \alpha + \sum \beta_i F_i + \varepsilon$$

where F_i are macroeconomic tradable security returns (for example, oil futures, regional stock indices, etc.). α represents the return not attributed to the tradables, and β_i are the respective weights of each factor return.

To estimate the weights in each period, we use a rolling training set and fit a Kalman filter method. Then, at each time step, we are able to forecast the next period's weights. The Kalman filter is a popular tool in fund replication to estimate the beta exposure of a fund against a set of liquid underlyings (Roncalli and Teiletche 2008). In addition, it has been shown to have better out-of-sample replication performance than rolling regressions with a fixed window, the other common replication technique.

The state-space formulation for the Kalman filter is as follows:

$$r_t^{index} = (r_t^{tradable})' w_t^{tradable} + S_t$$

$$w_t^{tradable} = w_{t-1}^{tradable} + V_t$$

$$S \sim N(0, R)$$

$$V \sim N(0, Q)$$

Using the Kalman filter, we attempt to replicate the returns of the state's assets with that of a basket of tradables, fitting the model using Maximum Likelihood Estimation. In order to capture as much of the components of a state's macroeconomic risks we include tradables from stock indices, bonds, commodities, precious metals as well as currency. In addition, to reduce the cost of our delta hedge, we focus our replication universe on liquid futures contracts. The following table shows the set of tradables we used for replication.

MXEA Index	CL1 Comdty	CADUSD Curncy
SPTR Index	Golds Comdty	GGGB1YR Index
MXEF Index	C 1 Comdty	GGGB10YR Index
DAX Index	W 1 Comdty	GGGB2YR Index
SX5E Index	S 1 Comdty	GDBR1 Index
TU1 Comdty	DXY Curncy	GDBR10 Index
TY1 Comdty	EUR Curncy	GDBR2 Index
US1 Comdty	JPYUSD Curncy	
USGG10yr index	GBP Curncy	
SPGSCI Index	AUD Curncy	

We tested this replication strategy on California's implied assets, and found a 98.2% correlation when using our tradables set. In addition, we found that the weights for the tradables were in line with our intuition of the sources of assets for California. Given the availability of data for Greece, a similar method can be applied.

By using this replication portfolio as a proxy for the state's assets, we can easily implement a method to control the risk associated with our model's delta by hedging with liquid tradable assets.

SECTION 5

CONCLUSION

The method detailed above is a rigorous quantitative procedure for providing a relative credit risk ranking to a sovereignty that is part of a monetary union. Our framework allows for us to build upon an established technique for calculating the probability of default for dominions not part of a monetary union and make intuitive extensions in order to adapt the technique to the monetary union situation.

Furthermore, our model does well in keying in on key differences between California and Greece; it allows for the default barrier to change when California has a smaller share of the United States debt, and permits for Greece to break away from the Euro Area. The model catches efficiently these sovereign events, even if they are very difficult to predict, by forming implied options on the assets of the sovereign countries. While our model should be tractable due to the several adjustable variables that depend on the region of the world and the type of economy, calibration may prove more difficult due to limited historical data. In our paper, we attempt to circumvent this issue by assuming that a certain economic event, such as a bailout or breakout, can have a non-negative probability.

Our hedging technique only uses tradable assets, and can be highly effective with sovereign assets. This methodology, while operating on non-tradable assets such as a state's implied assets, can be effectively applied in real markets by applying a replication methodology relying on the Kalman filter. This implies an ability to hedge without the inherent counterparty risk entrenched in buying a

CDS on a sovereign bond; our model focuses on hedging the default risk, not by buying insurance that may be itself risky.

Finally, our model could be improved by adding certain features like non-absorbing barriers for default, or additional factors for modeling political risk.

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